How Peer Effects Influence Energy Consumption

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Background

- Social comparisons influence people's behavior:
 - Conform to a standard
 - Receive social acclaim
 - Other people's choices can be informative (recommender systems)
- Network effects in social networks and platforms
 - Positive externalities
 - Impact of Peer Effects on energy consumption?¹
 - Various Randomized Controlled Trials (RCTs) to investigate such effects²
 - High consumers reduce most, efficient ones show "boomerang effect"
- Question
 - How can peer effects in energy networks be exploited for profit-maximization of the load serving entity?

Methodology



¹Hunt Allcott. "Social Norms and Energy Conservation". In: Journal of Public Economics 95.9 (2011), pp. 1082–1095.

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Consumers

• Set of consumers $\mathcal{I} = \{1, \dots, n\}$ with utility function

$$u_i = a_i x_i - b_i x_i^2 - p_i x_i + \gamma_i x_i \left(\sum_{j \in \mathcal{I}} w_{ij} x_j - x_i \right).$$

- Interaction matrix $W \in [0,1]^{n \times n}$
- Each user observes price p^{*}_i and x_{-i} and maximizes utility:

$$x_i^* = \arg \max_{x_i \ge 0} u_i(x_i, \mathbf{x}_{-i}, \gamma_i, W)$$

Load-Serving Entity

- Profit: $\Pi = \sum_{i \in \mathcal{I}} p_i x_i c_i x_i^2$
- Utility determines optimal price p^{*} to maximize Π
- Takes into account users' consumption decisions as a function of price **p**

$$\mathbf{p}^* = \arg \max_{\mathbf{p} \ge \mathbf{0}} \sum_{i \in \mathcal{I}} p_i x_i(p_i) - c_i x_i^2(p_i)$$

- Nash Equilibria of second stage game and first stage game
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Price and Consumption Equilibria

Perfect Price Discrimination

$$\mathbf{p}^* = \frac{\mathbf{a}}{2} + CZ \frac{\mathbf{a}}{2} - W^\top \Gamma Z \frac{\mathbf{a}}{4} + \Gamma WZ \frac{\mathbf{a}}{4},$$

$$\mathbf{x}^* = \left(C + B + 2\Gamma - \frac{W^\top \Gamma}{2} - \frac{\Gamma W}{2}\right)^{-1} \frac{\mathbf{a}}{2},$$

$$Z = \left[2\Gamma + B + C - \left(\frac{W^\top \Gamma}{2} + \frac{\Gamma W}{2}\right)\right]^{-1}.$$

$$\begin{array}{l} \begin{array}{l} \text{Single Price, Complete Information} \\ p_{u}^{*} = \left[1 - \frac{1^{\top}A^{-1}\mathbf{1}}{2\cdot\mathbf{1}^{\top}\left(A^{-1} + A^{-1}CA^{-1}\right)\mathbf{1}}\right]\bar{a}, \\ \mathbf{x}^{*} = A^{-1}\left[\mathbf{a} - \left(1 - \frac{1^{\top}A^{-1}\mathbf{1}}{2\cdot\mathbf{1}^{\top}\left(A^{-1} + A^{-1}CA^{-1}\right)\mathbf{1}}\right)\bar{a}\mathbf{1}\right], \\ A = B + 2\Gamma - \Gamma W, \qquad \bar{a} = \sum_{i=1}^{n} \bar{a}_{i}/n. \end{array}$$

$$\begin{split} & \overbrace{\tilde{\rho}_{u}^{*} \geq \frac{\mathbb{E}[a]}{2} \left[1 + \frac{c}{n} \mathbf{1}^{\top} \left[2\Gamma + (2\mathbb{E}[b] + c)I - \Gamma W \right]^{-1} \mathbf{1} \right],} \\ & \mathbb{E}[\tilde{x}_{j}] \geq \frac{\mathbb{E}[a] - \tilde{\rho}_{u, \text{LB}}^{*}}{n} \cdot \mathbf{1}^{\top} \left(2\Gamma + 2\mathbb{E}[b]I - \Gamma W \right)^{-1} \mathbf{1}. \end{split}$$

- $\bullet\,$ Complete knowledge of a and b
- Incentive for strongly influential users $W^{\top}\Gamma$
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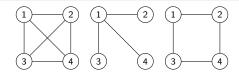
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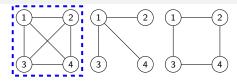
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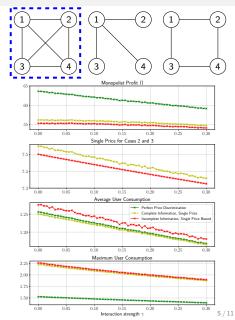
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$$\mathbf{p}^* = \frac{\mathbf{a}}{2} + CZ\frac{\mathbf{a}}{2} - W^{\top}\Gamma Z\frac{\mathbf{a}}{4} + \Gamma WZ\frac{\mathbf{a}}{4},$$

$$\mathbf{x}^* = \left(C + B + 2\Gamma - \frac{W^{\top}\Gamma}{2} - \frac{\Gamma W}{2}\right)^{-1}\frac{\mathbf{a}}{2},$$

$$Z = \left[2\Gamma + B + C - \left(\frac{W^{\top}\Gamma}{2} + \frac{\Gamma W}{2}\right)\right]^{-1}$$

$$\begin{split} & \frac{\mathsf{Single Price, Incomplete Information}}{\tilde{\rho}_u^* \geq \frac{\mathbb{E}[a]}{2} \left[1 + \frac{c}{n} \mathbf{1}^\top \left[2\Gamma + (2\mathbb{E}[b] + c)I - \Gamma W \right]^{-1} \mathbf{1} \right],} \\ & \mathbb{E}[\tilde{s}_I] \geq \frac{\mathbb{E}[a] - \tilde{\rho}_{u, \text{LB}}^*}{n} \cdot \mathbf{1}^\top \left(2\Gamma + 2\mathbb{E}[b]I - \Gamma W \right)^{-1} \mathbf{1}. \end{split}$$



Theorem (Monotonicity of Consumption Equilibrium)

If $a_i = a$, $b_i = b$, and $\gamma_i = \gamma \forall i \in I$, then x_i^* is strictly monotonically decreasing in γ independent of the network topology W.

Proof Sketch.

Take derivative $\frac{dx}{d\gamma} = -\frac{1}{4\gamma(b+\gamma)}K^{-1}F^{-1}(\mathbf{a} - \mathbf{p})$ and exploit diagonal dominance of K and F. Show that all elements $(K^{-1}F^{-1})_{ij}$ are positive.

Theorem (Influence of High Consumer)

Let $w_{ij} = \left(\sum_{j \in \mathcal{I}} \mathbf{1}_{w_{ij} > 0}\right)^{-1}$, $b_i = b$, $\gamma_i = \gamma$ and $a_i - p_i = \alpha$ for $\mathcal{N} = \{i \in \mathcal{I} \setminus j\}$. Let j be the "high" consumer. If $a_j - p_j = \tilde{\alpha} > n\alpha$, then for each neighbor i of j, x_i^* is initially increasing in γ , whereas x_j^* is strictly monotonically decreasing in γ .

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Theoretical Statements (cont'd.)

Theorem (Targeted Peer Effects)

For n = 2 users, the network effect reduces the sum of their consumptions iff

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The consumption equilibrium \mathbf{x}^* is inefficient as the social welfare S attained is suboptimal. Specifically, $x_i^* < x_i^\circ \forall i \in \mathcal{I}$, where \mathbf{x}° maximizes social welfare:

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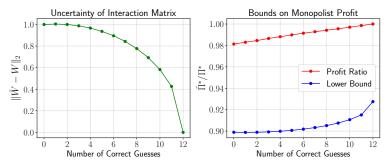
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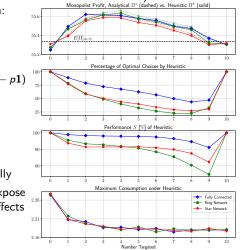
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Summary

- Setup of two-stage game-theoretic model for a network of electricity consumers
- Consumers seek to maximize individual utility function and derive utility from peer comparisons
- Investigated profit-maximizing pricing schemes (subgame-perfect equilibria)
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