## How Peer Effects Influence Energy Consumption

#### D.P. Zhou, M. Roozbehani, M.A. Dahleh, C.J. Tomlin

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### Background

- **•** Social comparisons influence people's behavior:
	- **Conform to a standard**
	- **B** Receive social acclaim
	- Other people's choices can be informative (recommender systems)
- Network effects in social networks and platforms
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- $\bullet$ 
	- Various Randomized Controlled Trials (RCTs) to investigate such effects<sup>2</sup>
	- High consumers reduce most, efficient ones show "boomerang effect"
- - How can peer effects in energy networks be exploited for profit-maximization of the

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#### Consumers

• Set of consumers  $\mathcal{I} = \{1, \ldots, n\}$  with utility function

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u_i = a_i x_i - b_i x_i^2 - p_i x_i + \gamma_i x_i \left( \sum_{j \in \mathcal{I}} w_{ij} x_j - x_i \right).
$$

- Interaction matrix  $W \in [0,1]^{n \times n}$
- Each user observes price  $p_i^*$  and  $\mathbf{x}_{-i}$  and maximizes utility:

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x_i^* = \arg\max_{x_i \geq 0} u_i(x_i, \mathbf{x}_{-i}, \gamma_i, W)
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- $\bullet$  Takes into account users' consumption decisions as a function of price  $\sf p$

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\mathbf{p}^* = \arg \max_{\mathbf{p} \geq 0} \sum_{i \in \mathcal{I}} p_i x_i(p_i) - c_i x_i^2(p_i)
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- $\bullet$  Can be determined with "backward induction"  $\qquad \qquad \bullet$  3/11

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### Price and Consumption Equilibria

Perfect Price Discrimination  
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\begin{aligned} \mathfrak{S} &\text{ingle Price, Complete Information} \\ \rho_u^* &= \left[1 - \frac{\mathbf{1}^\top A^{-1}\mathbf{1}}{2\cdot\mathbf{1}^\top\left(A^{-1} + A^{-1}CA^{-1}\right)\mathbf{1}}\right]\bar{\mathbf{a}}, \\ \mathbf{x}^* &= A^{-1}\left[\mathbf{a} - \left(1 - \frac{\mathbf{1}^\top A^{-1}\mathbf{1}}{2\cdot\mathbf{1}^\top\left(A^{-1} + A^{-1}CA^{-1}\right)\mathbf{1}}\right)\bar{\mathbf{a}}\mathbf{1}\right], \\ A &= B + 2\Gamma - \Gamma W, \qquad \bar{\mathbf{a}} = \sum_{i=1}^n a_i/n. \end{aligned}
$$

Single Price, Incomplete Information
$\tilde{p}_u^* \geq \frac{\mathbb{E}[a]}{2} \left[ 1 + \frac{c}{n} 1^\top [2\Gamma + (2\mathbb{E}[b] + c)I - \Gamma W]^{-1} 1 \right],$
$\mathbb{E}[\tilde{s}_i] \geq \frac{\mathbb{E}[a] - \tilde{p}_{u, \text{LB}}^*}{n} \cdot 1^\top (2\Gamma + 2\mathbb{E}[b]I - \Gamma W)^{-1} 1.$

- **Complete knowledge of a and b**
- **•** Incentive for strongly influential users  $W<sup>T</sup>$ Γ
- Additional cost for strongly influenced users ΓW

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A = B + 2\Gamma - \Gamma W, \qquad \mathbf{\vec{a}} = \sum_{i=1}^{n} a_{i}/n.
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$\hat{p}_u^* \geq \frac{\mathbb{E}[a]}{2} \left[ 1 + \frac{c}{n} \mathbf{1}^\top \left[ 2\Gamma + (2\mathbb{E}[b] + c)I - \Gamma W \right]^{-1} \mathbf{1} \right],$
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$\tilde{p}_u^* \geq \frac{\mathbb{E}[a]}{2} \left[ 1 + \frac{c}{n} \mathbf{1}^\top [2\Gamma + (2\mathbb{E}[b] + c)I - \Gamma W]^{-1} \mathbf{1} \right],$
$\mathbb{E}[\tilde{x}_i] \geq \frac{\mathbb{E}[a] - \tilde{p}_{u, \text{LB}}^*}{n} \cdot \mathbf{1}^\top (2\Gamma + 2\mathbb{E}[b]I - \Gamma W)^{-1} \mathbf{1}.$



Perfect Price Discrimination  
\n
$$
\mathbf{p}^* = \frac{\mathbf{a}}{2} + CZ\frac{\mathbf{a}}{2} - W^\top I Z\frac{\mathbf{a}}{4} + \Gamma W Z\frac{\mathbf{a}}{4},
$$
\n
$$
\mathbf{x}^* = \left(C + B + 2\Gamma - \frac{W^\top \Gamma}{2} - \frac{\Gamma W}{2}\right)^{-1} \frac{\mathbf{a}}{2},
$$
\n
$$
Z = \left[2\Gamma + B + C - \left(\frac{W^\top \Gamma}{2} + \frac{\Gamma W}{2}\right)\right]^{-1}.
$$

Single Price, Complete Information  
\n
$$
p_u^* = \left[1 - \frac{1^T A^{-1}1}{2 \cdot 1^T (A^{-1} + A^{-1}CA^{-1})1}\right] \bar{a},
$$
\n
$$
x^* = A^{-1} \left[a - \left(1 - \frac{1^T A^{-1}1}{2 \cdot 1^T (A^{-1} + A^{-1}CA^{-1})1}\right) \bar{a}1\right],
$$
\n
$$
A = B + 2\Gamma - \Gamma W, \qquad \bar{a} = \sum_{i=1}^n a_i/n.
$$

$$
\begin{aligned}\n\text{Single Price, Incomplete Information} \\
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#### Theorem (Monotonicity of Consumption Equilibrium)

If  $a_i = a$ ,  $b_i = b$ , and  $\gamma_i = \gamma \forall i \in \mathcal{I}$ , then  $x_i^*$  is strictly monotonically decreasing in  $\gamma$ independent of the network topology W .

Take derivative  $\frac{d\mathsf{x}}{d\gamma}=-\frac{1}{4\gamma(b+\gamma)}\mathsf{K}^{-1}\mathsf{F}^{-1}(\mathsf{a}-\mathsf{p})$  and exploit diagonal dominance of  $\mathsf K$  and

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#### Proof Sketch.

Take derivative  $\frac{d{\sf x}}{d\gamma}=-\frac{1}{4\gamma(b+\gamma)}{\sf K}^{-1}{\sf F}^{-1}({\sf a}-{\sf p})$  and exploit diagonal dominance of  ${\sf K}$  and F. Show that all elements  $(K^{-1}F^{-1})_{ij}$  are positive.

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Let  $w_{ij}=\left(\sum_{j\in\mathcal{I}}\mathbf{1}_{w_{ij}>0}\right)^{-1}$ ,  $b_i=b$ ,  $\gamma_i=\gamma$  and  $\mathsf{a}_i-\mathsf{p}_i=\alpha$  for  $\mathcal{N}=\{i\in\mathcal{I}\setminus j\}.$  Let  $j$  be the "high" consumer. If  $a_j - p_j = \bar{\alpha} > n\alpha$ , then for each neighbor i of j,  $x_i^*$  is initially increasing in  $\gamma$ , whereas  $x_j^*$  is strictly monotonically decreasing in  $\gamma$ .

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#### Proof Sketch.

Evaluate  $\frac{dx}{d\gamma}$  at  $\gamma = 0$  and use definition of peer effects.

### Theoretical Statements (cont'd.)

#### Theorem (Targeted Peer Effects)

For  $n = 2$  users, the network effect reduces the sum of their consumptions iff

$$
b_1 \leq \frac{(a_1-p)(4b_2+3\gamma)}{2(a_2-p)} \quad \text{and} \quad b_2 \leq \frac{(a_2-p)(4b_1+3\gamma)}{2(a_1-p)}.
$$

This can be generalized to  $n \geq 3$ .

Utility maximizing response of user *i* is 
$$
x_i^* = \frac{a_i - p_i + \gamma_i \sum_{j \in \mathcal{I}} w_{ij} x_j}{2(b_i + \gamma_i)}
$$
. Result follows

$$
\mathbf{x}^{\circ} = \left(C + \frac{B}{2} + \Gamma - \frac{W^{\top}\Gamma}{2} - \frac{\Gamma W}{2}\right)^{-1} \frac{\mathbf{a}}{2}
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#### Theorem (Efficiency)

The consumption equilibrium  $x^*$  is inefficient as the social welfare  $S$  attained is suboptimal. Specifically,  $x_i^* < x_i^{\circ}$   $\forall$   $i \in \mathcal{I}$ , where  $\mathbf{x}^{\circ}$  maximizes social welfare:

$$
\mathbf{x}^{\circ} = \left(C + \frac{B}{2} + \Gamma - \frac{W^{\top}\Gamma}{2} - \frac{\Gamma W}{2}\right)^{-1} \frac{\mathbf{a}}{2}
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.

Allocating users per-unit subsidies  $s_i = (b_i + \gamma_i) x_i^2/2$  can restore the social optimum.

### Network Uncertainty

### Unknown Network Structure

- Let  $W = W^{\top}$  and  $\Gamma = \gamma I$
- $\bullet$  Monopolist only has estimate  $\tilde{W}$ , where  $\tilde{W} = \tilde{W}^\top$
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$$
\frac{\tilde{\Pi}^*}{\Pi^*} \ge \frac{\lambda_{\min}(C + B + 2\Gamma - \Gamma W)}{\lambda_{\max}(C + B + 2\Gamma - \Gamma W) + \gamma \|W - \tilde{W}\|_2}
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- Which users should be exposed to peer effects to maximize profit?
- Assume p is exogenously set (by the Public Utilities Commission)
- **•** Formulate profit-maximizing problem:

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\begin{aligned}\n\text{maximize} & \quad \sum_{\delta_1,\dots,\delta_n}^n p_{\chi_i} - c_i x_i^2\\ \n\text{subject to} & \quad \mathbf{x} = (B + 2\Delta \Gamma - \Delta \Gamma W)^{-1} \left( \mathbf{a} - p \mathbf{1} \right) \\ \n\sum_{i=1}^n \delta_i &= m, \quad \delta_i \in \{0, 1\} \\ \n\Delta &= \text{diag}(\delta_1, \dots, \delta_n)\n\end{aligned}
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- Setup of two-stage game-theoretic model for a network of electricity consumers
- Consumers seek to maximize individual utility function and derive utility from peer
- Investigated profit-maximizing pricing schemes (subgame-perfect equilibria)
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# THANK YOU! QUESTIONS?