Eliciting Private User Information for Residential Demand Response

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December 12, 2017



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(Residential) Demand Response

• DR Provider seeks to collect "reductions" of electricity consumption from its customers under contract in exchange for monetary incentives



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Challenges

- How can reduction be measured?
- How heterogeneous are users in their reduction behavior?
- Can users "game" the system by misreporting their preferences?

End Users

- Each user $i \in \mathcal{I}$ has *estimated* baseline consumption $\hat{x}_i \in \mathbb{R}_+$ and actual, materialized consumption $x_i \in \mathbb{R}_+$
- Estimated reduction is $\delta_i = (\hat{x}_i x_i) \mathbf{1}_{i \in \mathcal{T}}$
- Demand curve / price elasticity of demand: $x_i(r_i) = \bar{x}_i \exp(-\alpha_i r_i)$
- User *i*'s utility function: $u_i = (r_i[\hat{x}_i x_i]_+ q_i[x_i \hat{x}_i]_+) \mathbf{1}_{i \in \mathcal{T}}$

$$\begin{array}{ll} \underset{r_{1},\ldots,r_{n}}{\text{minimize}} & \mathbb{E}_{\delta_{1},\ldots,\delta_{n}}\left[\sum_{i\in\mathcal{I}}\delta_{i}\left(r_{i}\mathbf{1}_{\delta_{i}<0}-q_{i}\mathbf{1}_{\delta_{i}\geq0}\right)\right] \\ \\ \text{subject to} & \mathbb{E}_{\delta_{1},\ldots,\delta_{n}}\left[\sum_{i\in\mathcal{I}}\delta_{i}\right]\geq M. \end{array}$$

- Incentivize subset of end-users $T \subset I$ with user-specific, per-unit rewards $\{r_i \in \mathbb{R}_+ \mid i \in T\}$
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At t = 0:

• User i's type is $m{ heta}_i=(lpha_i\sim F_lpha,m{\xi}_i\sim F_m{\xi})$, where $ar{x}_i\sim G_{m{\xi}_i}\sim G_{m{\xi}_i\sim F_m{\xi}}$

- Elasticity α_i characterizes willingness to reduce
- Base consumption \bar{x}_i is stochastic, does not follow rational profit-maximization

At t = 1

- Individual Rationality: $\mathbb{E}[u_i(f(m{ heta}_i, \mathbf{z}_{-i}))] \geq 0 \quad orall i \in \mathcal{I}, \; \mathbf{z} \in m{\Theta}$
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At t = 2 :

Social choice function f(z) consists of allocation and payment rule

At t = 3 :

- Settlements between DR Provider and users
- But: Not considered in Mechanism Design



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• Revelation Principle: Given a DSE, focus on *direct* mechanisms \rightarrow Users report their type truthfully $z_i = \theta_i$

Individual Rationality Constraints

• Expected payoff (user's utility) must be larger than any outside option:

$$\mathbb{E}[u_i(f(\boldsymbol{\theta}_i, \mathbf{z}_{-i}))] \geq \mathbb{E}[(r_i[\hat{x}_i - x_i]_+ - q_i[x_i - \hat{x}_i]_+) \mathbf{1}_{i \notin \mathcal{T}}] = \mathbf{0} \quad \forall i \in \mathcal{I}, \ \mathbf{z} \in \mathbf{\Theta}$$

=utility if not targeted



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Social Choice Function

- $f(\theta): \Theta \mapsto \mathcal{Y}$ maps type θ to collective choice $\mathbf{y} = (\mathbf{d}, \mathbf{r}) \in \mathcal{Y} = \{0, 1\}^n \times \mathbb{R}^n_+$
 - Vector of allocation decisions $\mathbf{d} \in \{0,1\}^n$
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VCG-Style Mechanism

Let μ_i(d_i = 1, r_i) = ∫_{ℝ+} u_i(α_i, r_i, x) dG_{ξi}(x) denote user i's expected utility, given reward r_i. Let r̃_i denote the unique r_i such that μ_i(d_i = 1, r̃_i) = 0.

$$\begin{aligned} j_{\max} &= \min_{j} \left\{ j \in \mathbb{N}_{+} \ \Big| \ \sum_{i=1}^{j} \delta_{i}(\tilde{r}_{j} | \boldsymbol{\theta}_{i}) \geq M \right\} \\ j(i) &= \min_{k} \left\{ k \in \mathbb{N}_{+} \ \Big| \ \sum_{s=1, s \neq i}^{k} \delta_{s}(\tilde{r}_{k} | \boldsymbol{\theta}_{s}) \geq M \right\} \qquad \forall \ i \in \{1, \dots, j_{\max}\} =: \mathcal{T} \\ r_{i} \leftarrow \tilde{r}_{j(i)} \geq \tilde{r}_{i} \quad \forall \ i \in \mathcal{T} \end{aligned}$$



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$$\begin{split} j_{\max} &= \min_{j} \left\{ j \in \mathbb{N}_{+} \ \Big| \ \sum_{i=1}^{j} \delta_{i}(\tilde{r}_{j} | \boldsymbol{\theta}_{i}) \geq M \right\} \\ j(i) &= \min_{k} \left\{ k \in \mathbb{N}_{+} \ \Big| \ \sum_{s=1, s \neq i}^{k} \delta_{s}(\tilde{r}_{k} | \boldsymbol{\theta}_{s}) \geq M \right\} \qquad \forall \ i \in \{1, \dots, j_{\max}\} =: \mathcal{T} \\ r_{i} \leftarrow \tilde{r}_{j(i)} \geq \tilde{r}_{i} \quad \forall \ i \in \mathcal{T} \end{split}$$

• Recall: $\theta_i = (\alpha_i \sim F_{\alpha}, \xi_i \sim F_{\xi})$, where $\bar{x}_i \sim G_{\xi_i} \sim G_{\xi_i \sim F_{\xi}}$, G lognormal

• *G* is parameterized by *shape*, *location*, *scale* parameters:

- Draw user types from hierarchical model
- n = 500 users, q = 5.0, $\alpha_i \sim {\sf unif}[0.05, 0.06]$
- Comparison to omniscient DR Provider

Omniscient DR Provider does not give up information rent

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Fundamental Problem of Causal Inference¹

- Either the outcome under treatment or under control is observed, but not both
- That is, the counterfactual consumption is always unobserved

• \hat{x}_i is an estimate of the counterfactual, prone to estimation inaccuracies CAISO 10-in-10 Baseline²

- Calculate \hat{x}_i as the mean of the 10 previous consumptions
- Reduction Components: $\delta_i = (\hat{x}_i \bar{x}_i) + \bar{x}_i(1 e^{-\alpha_i r_i}) =: \delta_i^{\mathsf{BL}} + \delta_i^r$
- Virtual Reductions due to variance in \hat{x}_i estimation

¹P. W. Holland. "Statistics and Causal Inference". In: Journal of the American Statistical Association 81.396 (1986), pp. 945–960.
² California Independent System Operator Corporation (CAISO): Fifth Replacement FERC Electric Tariff. 2014.

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Conclusion

Summary

- Modeled Residential Demand Response in Mechanism Design framework
- Intercept and slope of demand curve are users' private information
- DR Provider elicits private information with incentive compatible auction
- Practical Issue: "Baseline Gaming"

Future Work

- Improve baseline estimates (counterfactuals)
- Analyze serial correlation of consumption time series
- Extend one-shot problem to online, sequential auctions

THANK YOU! QUESTIONS?