Stability Analysis of Wholesale Electricity Markets under Dynamic Consumption Models and Real-Time Pricing

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Electricity Market Model

Electricity Suppliers $i \in \mathcal{S}$

- Convex, increasing cost function $c_i(\cdot) : \mathbb{R}_+ \mapsto \mathbb{R}_+$
- Given price λ, profit-maximizing production quantity is

$$s_i(\lambda) = \arg\max_{x \in \mathbb{R}_+} \lambda x - c_i(x) = \dot{c}_i^{-1}(\lambda)$$

Consumers $j \in \mathcal{D}$

- Traditionally[1]: *Static* utility function for consumption: $u_j = \arg\max_{x \in \mathbb{R}_+} v_j - \lambda x$
- *Today*: Time-varying model with memory $u_j(t) = f(\lambda(t), \lambda(t-1), u_j(t-1))$

Independent System Operator

- Solve Economic Dispatch Problem: Maximize social welfare in a network
- Subject to transmission, capacity, congestion constraints

$$\begin{aligned} \underset{\{u_j\}_{j \in \mathcal{D}}, \{s_i\}_{i \in \mathcal{S}}}{\text{maximize}} \quad & \sum_{j \in \mathcal{D}} v_j(u_j) - \sum_{i \in \mathcal{S}} c_i(s_i) \\ \text{subject to} \quad & \sum_{j \in \mathcal{D}} u_j = \sum_{i \in \mathcal{S}} s_i \end{aligned}$$

- Assumption: Absence of network constraints

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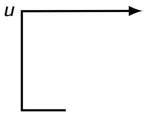

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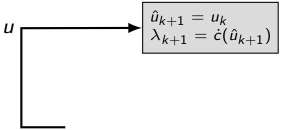

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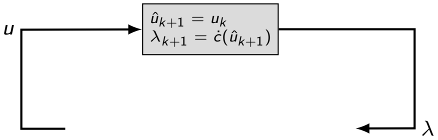

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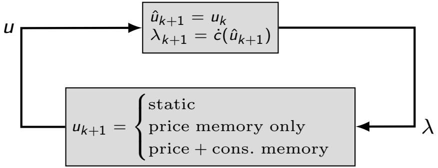
- ISO sets price $\lambda_{k+1} = \dot{c}(\hat{u}_{k-1})$, announces λ_{k+1} to producer
- Consumer decides on consumption u_{k+1}



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Electricity Consumption Model

Consumer's Inventory Problem[3]

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\begin{aligned}
\underset{u_0,\ldots,u_{n-1}}{\text{minimize}} \quad & \mathbb{E}_{\lambda_1,\ldots,\lambda_{n-1}}\left[\sum_{k=0}^{n-1} \lambda_k u_k + p(x_{k+1}) + h(u_k, d_k)\right] \\
\text{subject to} \quad & x_{k+1} = x_k + u_k - d_k \\
& x_k \leq 0 \\
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- Actual consumption: u_k

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Assumption

- h and p convex in first argument

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Electricity Consumption Model – Solution

Bellman Equation and Dynamic Programming

$$J_k^* = \min_{u_k} \lambda_k u_k + p(x_{k+1}) + h(u_k, d_k) + \mathbb{E}_{\lambda_{k+1},\ldots,\lambda_{n-1}} \left[J_{k+1}^* \right].$$

$\Rightarrow$ Solution u_k^* is function of previous prices and consumption decisions

Case $h \equiv 0$:

$$u_{n-k}^* = d_{n-k} - \dot{p}^{-1} \left(\frac{\lambda_{n-k} - \lambda_{n-k-1}}{k-1} \right), \quad k = 2, \ldots, n$$

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Assumptions

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Electricity Consumption Model – Solution

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Analysis of Market Stability

Case 1: Static Consumption Model

- Utility and cost functions of consumer and producer are constant and time-invariant[4]

Case 2: Consumption Model with Price Memory

$$\lambda_{k+1} = \dot{c}(\hat{u}_{k+1}) = \dot{c}(u_k) = \dot{c}\left(\dot{p}^{-1}(-\lambda_k) - \dot{p}^{-1}(-\lambda_{k-1}) + d_k\right)$$

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$$c(x) = \alpha x^2 \qquad p(x) = \beta x^2$$

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- System of linear, non-homogeneous difference equations
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[4] Mardavij Roozbehani, Munther Dahleh, and Sanjoy K. Mitter. "Volatiliy of Power Grids under Real-Time Pricing". In: *IEEE Transactions on Power Systems* (2012).

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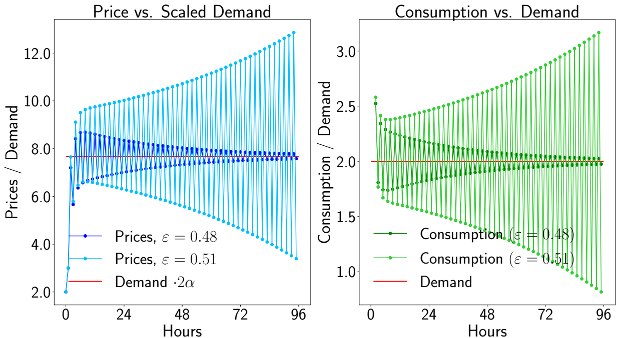
Simulation for Constant Demand $d_k = d$



Parameters Used:

- $d = 2$
- $\beta = 4$
- $\alpha = 1.92$ or $\alpha = 2.04$
- Initial conditions: $\lambda_0 = \lambda_1 = 3$

Analysis of Market Stability (cont'd.)

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Variable Demand Model

- Sinusoid of period 12 hours[5]: $2\alpha d_k = \mu + A \sin((k-5)\pi/6)$

[5] D. Zhou, M. Balandat, and C. Tomlin. "Residential Demand Response Targeting Using Machine Learning with Observational Data". In: *55th IEEE Conference on Decision and Control* (2016).

Analysis of Market Stability (cont'd.)

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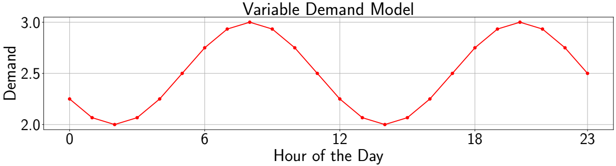

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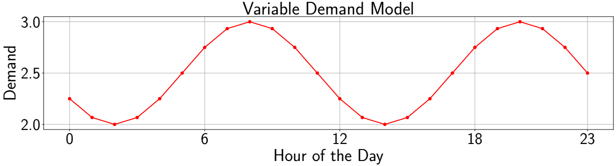

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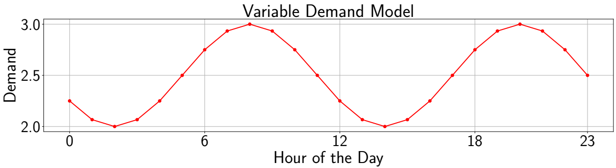

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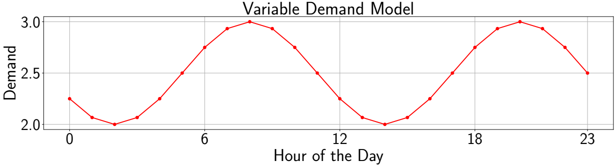

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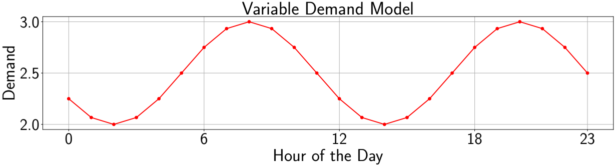

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Analysis of Market Stability (cont'd.)

Case 3: Consumption Model with Price and Consumption Memory

- Difference Equation for prices:

$$\lambda_{k+1} = \frac{\rho - \alpha}{\gamma + \rho}\lambda_k + \frac{\alpha}{\gamma + \rho}\lambda_{k-1} + 2\alpha d_k - \frac{2\alpha\rho}{\gamma + \rho} d_{k-1}$$

- Recall assumptions made:
  - $V_k(x_k) = p(x_k) + J_k^* = \gamma x_k^2$
  - $h(u_k, d_k) = \rho(u_k - d_k)^2$
- Stability guaranteed for $\tilde{\varepsilon} := \alpha/\gamma < 1/2 + \rho/\gamma$
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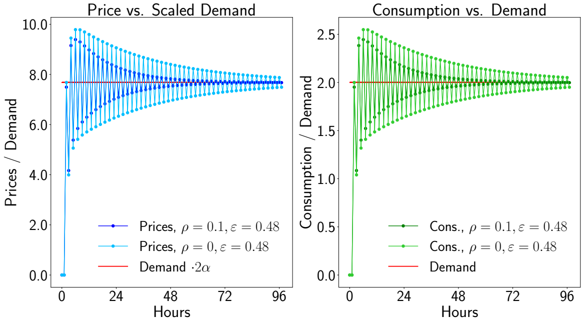

Parameters Used

- $\alpha = 2.04$
- $\gamma = 4$
- $\rho = 0$ or $\rho = 0.1$
- Initial conditions: $\lambda_0 = \lambda_1 = 0$

Summary

Real-Time Pricing of Electricity

- RT Pricing relays risk to end-use customers
- Implications of RT Pricing not well understood
- *Our contribution*: Analysis of price and consumption stability

Dynamic Consumption Behavior

- Previous analysis assumed *static* utility functions
- *Our contribution*: Derivation of consumption models with price and consumption memory

Follow-Up Work

- Formulation of *hedging strategies* for load-serving entities to mitigate risk

Summary

Real-Time Pricing of Electricity

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- Implications of RT Pricing not well understood
- *Our contribution*: Analysis of price and consumption stability

Dynamic Consumption Behavior

- Previous analysis assumed *static* utility functions
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Follow-Up Work

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Thank You!
Questions?